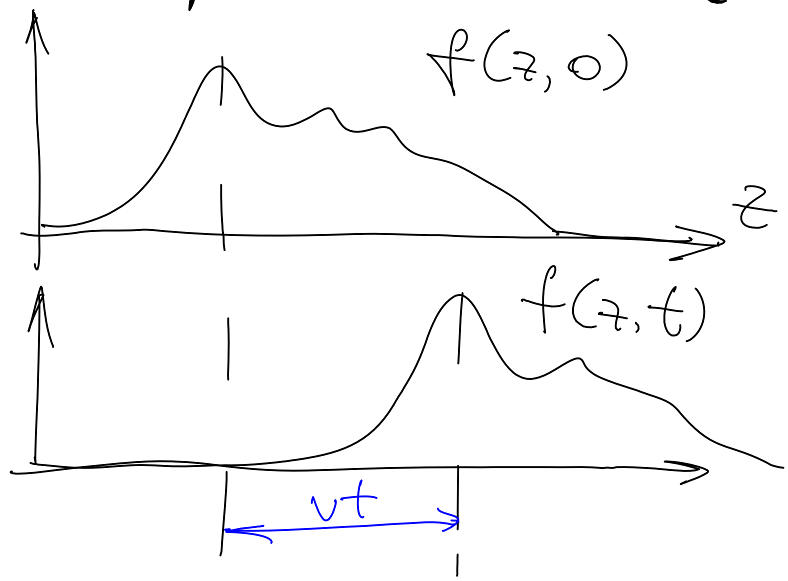
Physics 100C

Monday, March 30, 2009

Lecture #1

Waves, Brief Summary (1D)



If $f(z,t)$ can be represented as a function of $u = z - vt$:

$$f(z,t) = g(z - vt)$$

it represents a wave of constant shape, traveling at velocity v (along $\vec{z}$ direction)

For many systems wave equation has differential form:

$$\frac{\partial f}{\partial z^2} = \frac{1}{v^2} \cdot \frac{\partial^2 f}{\partial t^2} \quad \text{(1D version)}$$

See pp. 365-366 for example of string under tension

of string under tension

Allows solutions:

$$f(z,t) = g(u) = g(z-vt)$$

$$\frac{\partial f}{\partial z} = \frac{\partial g}{\partial u} \cdot \underbrace{\frac{\partial u}{\partial z}}_{=1} = \frac{\partial g}{\partial u}$$

$$\frac{\partial f}{\partial t} = \frac{\partial g}{\partial u} \cdot \underbrace{\frac{\partial u}{\partial t}}_{=-v} = -v \frac{\partial g}{\partial u}$$

$$\frac{\partial^2 f}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial g}{\partial u}\right) = \frac{\partial^2 g}{\partial u^2}$$

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial t}\left(-v\frac{\partial g}{\partial u}\right) = v^2 \frac{\partial g}{\partial u^2}$$

So that: $\frac{\partial^2 g}{\partial u^2} = \frac{1}{v^2} \cdot \frac{\partial^2 f}{\partial t^2}$

Sinusoidal waves:

$$f(z,t) = A \cdot \cos\left(k(z-vt)+\delta\right)$$

$k = \frac{2\pi}{\lambda}$ "wavenumber" (so that when $z-vt$ is incremented by $n\lambda$, phase shift is $= 2\pi n$)

Also: $T = \frac{\lambda}{} = \frac{1}{}$

Also: $T = \frac{\lambda}{v} = \frac{1}{\nu}$

$$\omega = \frac{2\pi}{T} = 2\pi\nu = k \cdot v$$

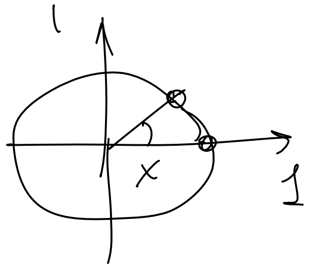
$$f(z,t) = A \cdot \cos(kz - \omega t + \delta)$$

wave traveling to the right (along $\vec{z}$)

For waves traveling left?
CHANGE sign of k OR ω.

Complex notation:

$$e^{ix} = \cos x + i \sin x$$



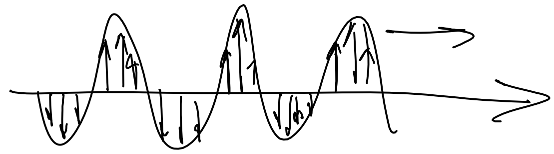
e.g. $e^{i\pi} = -1$

$$f(z,t) = \text{Re}\left[\tilde{A} \cdot e^{i(kz - \omega t)}\right]$$

very useful (see Ex. 9.1)

Longitudinal:

Transverse:



EM in vacuum:

Maxwell equations:

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} = 0 \qquad \text{Gauss}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \text{NO MAGN. MONOPOLES}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \text{FARADAY}$$

$$\vec{\nabla}\times\vec{B} = \underbrace{\cancel{\mu_0\vec{J}}}_{0} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} \qquad \text{Ampere (+MAXWELL)}$$

Coupled 1st ORDER

CURL IT AWAY:

$$\nabla\times(\nabla\times E) = \nabla(\underbrace{\cancel{\nabla\cdot E}}_{0}) - \nabla^2 E = -\nabla^2 E =$$

$$= \nabla\times\left(-\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla\times B) = -\mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}$$

$$\nabla\times(\nabla\times B) = \nabla(\underbrace{\cancel{\nabla\cdot B}}_{0}) - \nabla^2 B = -\nabla^2 B =$$

$$= \nabla\times\left(\mu_0\varepsilon_0\frac{\partial E}{\partial t}\right) = \mu_0\varepsilon_0\frac{\partial}{\partial t}(\nabla\times E) =$$

$$= -\mu_0\varepsilon_0\cdot\frac{\partial^2 B}{\partial t^2}$$

Therefore:

$$\boxed{\begin{aligned} \nabla^2 E &= \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \\ \nabla^2 B &= \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} \end{aligned}}$$

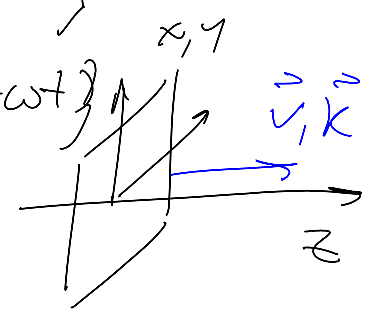
3D wave equations! with velocity $\boxed{c^2 = \frac{1}{\mu_0 \varepsilon_0}}$ SPEED OF LIGHT IN VACUUM $3 \cdot 10^8\ m/s$

Monochromatic Plane Waves

Waves traveling along z, have NO x-y dependence:

$$\vec{E}(z,t) = \mathrm{Re}\left\{ \tilde{E}_0 \cdot e^{i(kz-\omega t)} \right\}$$

$$\vec{B}(z,t) = \mathrm{Re}\left\{ B_0 \cdot e^{i(kz-\omega t)} \right\}$$



Gauss Law:

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\underbrace{\frac{\partial E_x}{\partial x}}_{=0} + \underbrace{\frac{\partial E_y}{\partial y}}_{=0} + \frac{\partial E_z}{\partial z} = 0$$

(because no x-y dependence)

(because no x-y dependence)

$$\frac{\partial E_z}{\partial z} = ik E_0^z = 0 \Rightarrow E_0^z = 0$$

Similarly $B_0^z = 0$

EM waves are <u>transverse</u>!

$\vec{E}$ & $\vec{B}$ vectors $\perp$ propagation

Faraday Law:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & 0 \end{vmatrix} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Downarrow$$

Along $\hat{x}$:

$$-\hat{x}\frac{\partial}{\partial z} E_y = -\hat{x}\cdot\frac{\partial B_x}{\partial t}$$

$$ik E_0^y = -i\omega B_0^x \Rightarrow B_0^x = -\frac{k}{\omega} E_0^y$$

Along $\hat{y}$:

$$\hat{y}\frac{\partial}{\partial z} E_x = -\hat{y}\cdot\frac{\partial B_y}{\partial t}$$

$$ik E_0^x = -i\omega B_0^y \Rightarrow B_0^y = \frac{k}{\omega} E_0^x$$

$$\underset{\partial z}{ik E_0^x} = \underset{\partial t}{i\omega B_0^y} \Rightarrow B_0^y = \frac{k}{\omega} E_0^x$$

If we pick $\hat{x}$ so that its along B_0; i.e. $B_0^y = 0$

then $E_0^x = 0$ as well

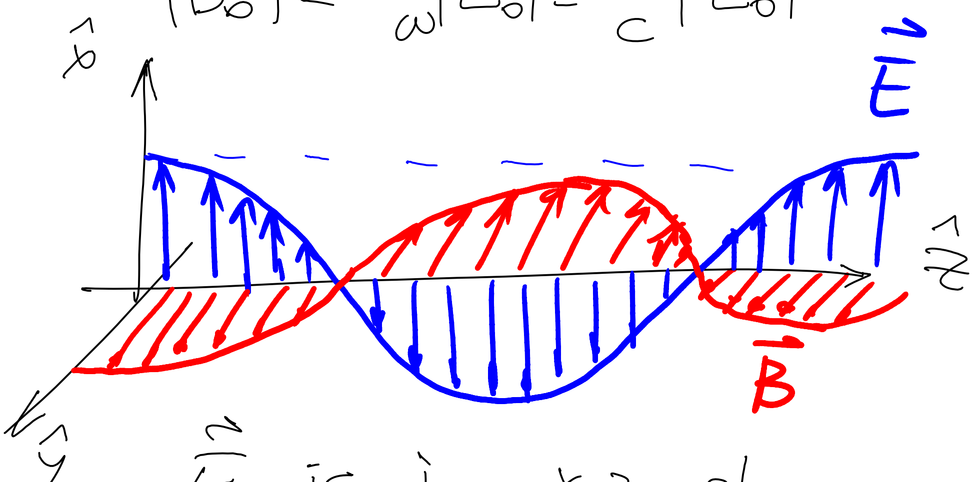
and $\vec{E}_0 \parallel \hat{y}$, $\vec{B}_0 \parallel \hat{x}$,

$\vec{E}_0 \perp \vec{B}_0$

OR: $\vec{B}_0 = \frac{k}{\omega}(\hat{z} \times \vec{E}_0)$

$\vec{E}_0$ and $\vec{B}_0$ in phase, and perpendicular to each other

$$|\vec{B}_0| = \frac{k}{\omega}|\vec{E}_0| = \frac{1}{c}|\vec{E}_0|$$



$\vec{E}$ is in xz plane

$\vec{B}$ is in yz plane

*** FOR JEFF:** (Since he asked)

What about $\hat{z}$ component?

$$\hat{z}\frac{\partial}{\partial x}E_y - \hat{z}\frac{\partial}{\partial y}\cdot E_x = -\frac{\partial B_z}{\partial t}\hat{z}$$

$$B_z = 0 \; ; \quad \frac{\partial E_y}{\partial x} = \frac{\partial E_x}{\partial y} = 0$$

because waves are PLANE

(no x-y dependence)

* Also: Ampere-Maxwell Eq. does not produce any new info Solution above already satisfies.